JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON FRIDAY 24th JANUARY 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a}.\vec{c} = 5$, then $|\vec{c}|$ is equal to

(1) $\frac{1}{3\sqrt{2}}$ (2) 18

(3) 16 (4) $\sqrt{\frac{11}{6}}$

Ans. (4)

Sol. $\vec{c} = \lambda(\vec{b}\times(\vec{a}\times\vec{b}))$

$= \lambda((\vec{b}.\vec{b})\vec{a} - (\vec{a}.\vec{b})\vec{b})$

$= \lambda(11\vec{a} - 2\vec{b}) = \lambda(11i + 22j + 33k - 6i - 2j + 2k)$

$= \lambda(5i + 20j + 35k)$

$= 5\lambda(5i + 4j + 7k)$

= Given $\vec{c}.\vec{a} = 5$

$= 5\lambda(1 + 8 + 21) = 5 = \lambda = \frac{1}{30} \Rightarrow \vec{c} = \frac{1}{6}(i + 4j + 7k)$

$|\vec{c}| = \frac{\sqrt{1+16+49}}{6} = \sqrt{\frac{11}{6}}$

2. In $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0, then I(9, 14) + I(10, 13) is

(1) I(9, 1) (2) I(19, 27)

(3) I(1, 13) (4) I(9, 13)

Ans. (4)

Sol. $I(m,m) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

Let $x = \sin^2\theta$ $dx = 2\sin\theta\cos\theta d\theta$

$I(m,n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1}d\theta$

$I(9, 14) + I(10, 13) = 2\int_0^{\pi/2} (\sin\theta)^{17}(\cos\theta)^{27}d\theta$

$+2\int_0^{\pi/2} (\sin\theta)^{19}(\cos\theta)^{25}d\theta$

$= 2\int_0^{\pi/2} (\sin\theta)^{17}(\cos\theta)^{25}d\theta$

$= I(9, 13)$

3. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function such that $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$. If the $\lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta$; $\alpha, \beta \in \mathbb{R}$, then $\alpha + 2\beta$ is equal to

(1) 3 (2) 5

(3) 4 (4) 6

Ans. (3)

Sol. $F(x) - 6f(1/x) = \frac{35}{3x} - \frac{5}{2}$ ……(1)

Replace $x \to \frac{1}{x}$

$F(1/x) - 6(x) = \frac{35x}{3} - \frac{5}{2}$ ……(2)

Using (1) & (2)

$f(x) = -2x - \frac{1}{3x} + \frac{1}{2}$

$B = \lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right)$

$= \lim_{x\to 0}\left(\frac{1}{\alpha x} - 2x - \frac{1}{3x} + \frac{1}{2}\right)$

$\alpha = 3$, $B = ½$

So, $\alpha + 2B = 3 + 1 = 4$

4. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term –p and common difference p is $\sqrt{2026 S_{2025}}$, then the absolute difference between 20th and 15th terms of the A.P. is

(1) 25 (2) 90

(3) 20 (4) 45

Ans. (1)

Sol. $Sn = \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}$ …. N terms

$S_{2025} = \sum_{n=1}^{2025}\frac{1}{n(n+1)} = \sum_{n=1}^{2025}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$= \left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)$ …..$\left(\frac{1}{2025}-\frac{1}{2026}\right)$

$= \frac{2025}{2026}$

$\sqrt{2026.S_{2025}} = \sqrt{2025} = 45$

Given : $\frac{6}{2}\left[-2p+(6-1)p\right]=45$

$9p = 45$

$p = 5$

$|A_{20} - A_{15}| = |-5 + 19 \times 5| - [-5 + 14 \times 5]$

$= |90 - 65|$

$= 25$

5. Let $f(x) = \frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+...+f\left(\frac{59}{15}\right)\right)$ is equal to

(1) 118 (2) 92

(3) 102 (4) 108

Ans. (1)

Sol. $f(x) = \frac{42^x+16}{2.2^{2x}+16.2^x+32}$

$f(x) = \frac{2\left(2^x+4\right)}{2^{2x}+8.2^x+16}$

$f(x) = \frac{2}{2^x+4}$

$f(4-x) = \frac{2^x}{2\left(2^x+4\right)}$

$f(x) + f(4-x) = \frac{1}{2}$

So, $f\left(\frac{1}{15}\right)+f\left(\frac{59}{15}\right)=\frac{1}{2}$

Similarly = $f\left(\frac{29}{15}\right)+f\left(\frac{31}{15}\right)=\frac{1}{2}$

$f\left(\frac{30}{15}\right)=f(2)=\frac{2}{2^2+4}=\frac{2}{8}=\frac{1}{4}$

$\Rightarrow 8\left(29\times\frac{1}{2}+\frac{1}{4}\right)$

Ans. 118

Option (4)

6. If α and β are the roots of the equation $2z^2 - 3z - 2i = 0$, where $i=\sqrt{-1}$, then $16.\text{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right).\text{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to

(1) 398 (2) 312

(3) 409 (4) 441

Ans. (4)

Sol. $2z^2-32-2i=0$

$2\left(z-\frac{i}{z}\right)=3$

$\alpha-\frac{i}{\alpha}=\frac{3}{2}$

$\Rightarrow \alpha^2-\frac{1}{\alpha^2}-2i=\frac{9}{4}$

$\Rightarrow \alpha^2-\frac{1}{\alpha^2}-2i=\frac{9}{4}$

$\Rightarrow \frac{9}{4}+2i=\alpha^2-\frac{1}{\alpha^2}$

$\Rightarrow \frac{81}{16}-4+9i=\alpha^4+\frac{1}{\alpha^4}-2$

$\Rightarrow \frac{49}{16}+9i=\alpha^4+\frac{1}{\alpha^4}$

Similarly

$\Rightarrow \frac{49}{16}+9i=\beta^4+\frac{1}{\beta^4}$

$\Rightarrow \frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}=\frac{\alpha^{15}\left(\alpha^4+\frac{1}{\alpha^4}\right)+\beta^{15}\left(\beta^4+\frac{1}{\beta^4}\right)}{\alpha^{15}+\beta^{15}}$

$=\frac{\left(\alpha^{15}+\beta^{15}\right)\left(\frac{49}{16}+9i\right)}{\left(\alpha^{15}+\beta^{15}\right)}$

Real = $\frac{49}{16}$

Im = 9

Ans. 441

7. $\lim_{x\to 0} \text{cosec}x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$ is

(1) 0 (2) $\frac{1}{2\sqrt{5}}$

(3) $\frac{1}{\sqrt{15}}$ (4) $-\frac{1}{2\sqrt{5}}$

Ans. (4)

Sol. $\lim_{x\to 0} \text{cosec}x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$

$$\lim_{x\to 0}\frac{\text{cosec}x\left(\cos^2 x+3\cos x-\sin x-4\right)}{\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{1}{\sin x}\frac{\left(\cos^2 x+3\cos x-4\right)-\sin x}{\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{(\cos x+4)(\cos x-1)-\sin x}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}(\cos x+4)-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{-\left(\sin\frac{x}{2}(\cos x+4)+\cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$-\frac{1}{2\sqrt{5}}$$

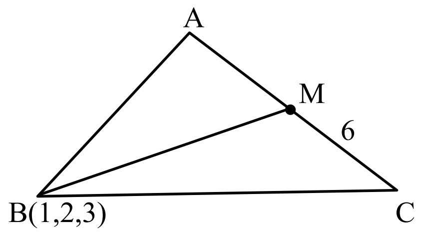
8. Let in a ΔABC, the length of the side AC be 6, the vertex B be (1, 2, 3) and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of ΔABC is

(1) 42 (2) 21

(3) 56 (4) 17

Ans. (2)

Sol.



Let M $(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$\overrightarrow{BM}=(3\lambda+5)\hat{i}+(2\lambda+5)\hat{j}+(-2\lambda+4)\hat{k}$

$\overrightarrow{AC}.\overrightarrow{BM}=0=3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$

$\overrightarrow{BM}=2\hat{i}+3\hat{j}+6\hat{k}$

$|\overrightarrow{BM}|=7$

Area = $\frac{1}{2}\times 6\times 7 = 21$

Option (2)

9. Let $y = y(x)$ be the solution of the differential equation $\left(xy-5x^2\sqrt{1+x^2}\right)dx + (1 + x^2)dy = 0$, $y(0) = 0$. Then $y\left(\sqrt{3}\right)$ is equal to

(1) $\frac{5\sqrt{3}}{2}$ (2) $\sqrt{\frac{14}{3}}$

(3) $2\sqrt{2}$ (4) $\sqrt{\frac{15}{2}}$

Ans. (1)

Sol. $\left(1+x^2\right)\frac{dy}{dx}+xy=5x^1\sqrt{1+x^2}$

$$\frac{dy}{dx}+\frac{xy}{1+x^2}=\frac{5x^2}{\sqrt{1+x^2}}$$

$\therefore$ I.F. = $e^{\int\frac{x}{1+x^2}dx}=e^{\frac{\ln(1+x^2)}{2}}=\sqrt{1+x^2}$

$\therefore\ y\sqrt{1+x^2}=\int\frac{5x^2}{\sqrt{1+x^2}}.\sqrt{1+x^2}dx$

$\therefore\ y\sqrt{1+x^2}=\int\frac{5x^2}{\sqrt{1+x^2}}.\sqrt{1+x^2}dx$

$y\sqrt{1+x^2}=\frac{5x^3}{3}+C$

$\because$ $y(0) = 0 \Rightarrow 0 = 0 + C \Rightarrow C = 0$

$\therefore\ y=\frac{5x^3}{3\sqrt{1+x^2}}$

$y\left(\sqrt{3}\right)=\frac{15\sqrt{3}}{32} = \boxed{\frac{5\sqrt{3}}{2}}$

Option (1)

10. Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b), be $\frac{7}{4}$.

Then the absolute difference of the eccentricities of two such ellipses is

(1) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$ (2) $\frac{1-\sqrt{3}}{\sqrt{2}}$

(3) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$ (4) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

Ans. (3)

Sol. Product of focal distances = $(a + ex_1)(a - ex_1)$

$= a^2 - e^2x_1^2 = a^2 - e^2(3)$

$= a^2 - 3e^2 = \frac{7}{4} \Rightarrow a^2 = \frac{7}{4} + 3e^2$

$\Rightarrow 4a^2 = 7 + 12e^2$

& $\left(\sqrt{3}, \frac{1}{2}\right)$ lines on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \frac{3}{a^2} + \frac{1}{4b^2} = 1$

$\frac{3}{a^2} + \frac{1}{4(a^2)(1-e^2)} = 1$

$12(1-e^2) + 1 = 4a^2(1-e^2)$

$13 - 12e^2 = (7 + 12e^2)(1-e^2)$

$\Rightarrow 13 - 12e^2 = 7 - 7e^2 + 12e^2 - 12e^4$

$\Rightarrow 12e^4 - 17e^2 + 6 = 0$

$\therefore e^2 = \frac{17 \pm \sqrt{289-288}}{24} = \frac{17 \pm 1}{24} = \frac{3}{4} \& \frac{2}{3}$

$\therefore e = \frac{\sqrt{3}}{2} \& \sqrt{\frac{2}{3}}$

$\therefore$ difference $= = \frac{\sqrt{3}}{2} - \sqrt{\frac{2}{3}} = \frac{3-2\sqrt{2}}{2\sqrt{3}}$

Option (3)

11. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5. The probability, that A wins if A makes the first throw, is

(1) $\frac{9}{17}$ (2) $\frac{9}{19}$

(3) $\frac{8}{17}$ (4) $\frac{8}{19}$

Ans. (2)

Sol. $p(S_5) = \frac{1}{9}$

$p(S_5) = \frac{5}{36}$

required prob $= \frac{1}{9} + \frac{8}{9}.\frac{31}{36}.\frac{1}{9} + \left(\frac{8}{9}.\frac{31}{36}\right)^2.\frac{1}{9} + \ldots\infty$

$= \frac{\frac{1}{9}}{1-\frac{62}{81}} = \frac{9}{19}$

Option(2)

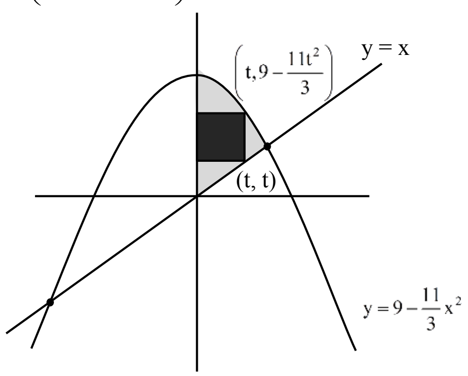
12. Consider the region $R = \left\{(x,y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\right\}$. The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is :

(1) $\frac{625}{111}$ (2) $\frac{730}{119}$

(3) $\frac{567}{121}$ (4) $\frac{821}{123}$

Ans. (3)

Sol. $t.\left(9 - \frac{11t^2}{3} - t\right)$



$$A = 9t - t^2 - \frac{11}{3}t^3$$

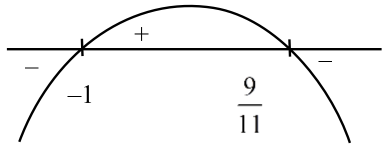
$$\frac{dA}{dt} = 9 - 2t - 11t^2$$

$\Rightarrow 11t^2 + 2t - 9 = 0$

$11t^2 + 11t - 9t - 9 = 0$

$t = -1$ & $t = \frac{9}{11}$

$\therefore \frac{dA}{dt} =$



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Maxima at $t = \frac{9}{11}$

$$\therefore \text{ largest area} = \frac{9}{11}\left(9 - \frac{11}{3}.\frac{81}{121} - \frac{9}{11}\right)$$

$$= \frac{9}{11}.\frac{63}{11} = \frac{567}{121}$$

Option (3)

13. The area of the region $\{(x, y) : x^2 + 4x + 2 \le y \le |x + 2|\}$ is equal to

(1) 7 (2) 24/5

(3) 20/3 (4) 5

Ans. **(3)**

Sol. $x^2 + 4x + 2 \le y \le |x + 2|$

The area bounded between

$y = x^2 + 4x + 2 = (x + 2)^2 - 2$

and $y = |x + 2|$ is same as

area bounded between $y = x^2 - 2$ and $y = |x|$

For P.O.I $|x|^2 - 2 = |x|$

$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$

$$\therefore \text{ Required area} = -\int_{-2}^{2}(x^2 - 2)dx + \int_{-2}^{2}|x|\,dx$$

$$= -2\int_0^2 (x^2 - 2)dx + 2\int_0^2 x.dx$$

$$= -2\left[\frac{x^3}{3} - 2x\right]_0^2 + 2\left[\frac{x^2}{2}\right]_0^2$$

$$= -2\left[\frac{8}{3} - 4\right] + 2\left[\frac{4}{2}\right]$$

$$= -2 \times \left(\frac{-4}{3}\right) + 4$$

$$= \frac{20}{3}$$

14. For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$.

He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is

(1) 7 (2) 4

(3) 9 (4) 5

Ans. **(1)**

Sol. Mean $\bar{x} = 5.5$

$$= \sum_{i=1}^{10} x_i = 5.5 \times 10 = 55$$

$$= \sum_{i=1}^{10} x_i^2 = 371$$

$$\left(\sum x_i\right)_{new} = 55 - (4+5) + (6+8) = 60$$

$$\left(\sum x_i\right)_{new} = 371 - (4^2 + 5^2) + (6^2 + 8^2) = 430$$

$$\text{Variance } \sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$$

$$\sigma^2 = \frac{430}{10} - \left(\frac{60}{10}\right)^2$$

$\sigma^2 = 43 - 36$

$\sigma^2 = 7$

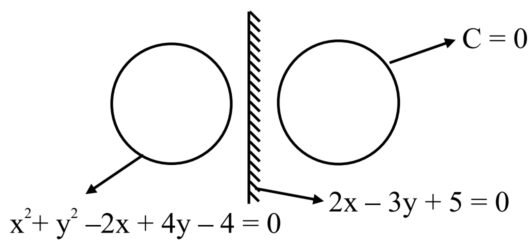
15. Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the arc AB is $(1/6)^{th}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to

(1) 3 (2) $3 + \sqrt{3}$

(3) $4 - \sqrt{3}$ (4) 4

Ans. **(4)**

Sol.



Centre (1, –2), r = 3

Reflection of (1, –2) about 2x – 3y + 5 = 0

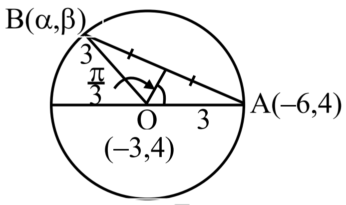
$$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{-2(2+6+5)}{13}=-2$$

x = –3, y = 4

Equation of circle 'C'

C : $(x+3)^2 + (y-4)^2 = 9$

A.T.Q.



AB= $3\sqrt{3}$

$$\ell(\text{arcAB}) = \frac{1}{6}\times 2\pi r$$

$$r\theta = \frac{1}{6}\times 2\pi r$$

$$\theta = \frac{\pi}{3}$$

$(\alpha + 6)^2 + (\beta - 4)^2 = 27$

$(\alpha + 3)^2 \pm (\beta - 4)^2 = 9$

$(\alpha + 6)^2 - (\alpha + 3)^2 = 18$

$\Rightarrow 6\alpha = -9$

$\Rightarrow \boxed{\alpha = \frac{-3}{2}}, \boxed{\beta = \left(4 - \frac{3\sqrt{3}}{2}\right)}$

$\therefore\ \beta - \sqrt{3}\alpha$

$\left(4 - \frac{3\sqrt{3}}{2}\right) + \frac{3\sqrt{3}}{2}$

= 4

16. For some n ≠ 10, let the coefficients of the 5th, 6th and 7th terms in the binomial expansion of $(1 + x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1 + x)^{n+4}$ is :

(1) 70 (2) 35

(3) 20 (4) 10

Ans. **(2)**

Sol. $(1 + x)^{n+4}$

${}^{n+4}C_4, {}^{n+4}C_5, {}^{n+4}C_6, \rightarrow$ A.P.

$\Rightarrow 2 \times {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6$

$\Rightarrow 4 \times {}^{n+4}C_5 = ({}^{n+4}C_4 + {}^{n+4}C_5) + ({}^{n+4}C_5 + {}^{n+4}C_6)$

$\Rightarrow 4 \times {}^{n+4}C_5 = {}^{n+5}C_5 + {}^{n+5}C_6$

$$\Rightarrow 4\times\frac{(n+4)!}{5!.(n-1)!}=\frac{(n+6)!}{6!.n!}$$

$$\Rightarrow\ 4=\frac{(n+6)(n+5)}{6n}$$

$\Rightarrow n^2 + 11n + 30 = 24\,n$

$\Rightarrow n^2 - 13\,n + 30 = 0$

$\Rightarrow$ n = 3, 10(rejected)

$\because\ n \neq 10$

$\therefore$ Largest binomial coefficient in expansion of $(1 + x)^7$

$(\because\ n + 4 = 7)$

is coeff. of middle term

$\Rightarrow {}^7C_4 = {}^7C_3 = 35$

N.T.A. Ans Option (2)

17. The product of all the rational roots of the equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$, is equal to :

(1) 14 (2) 7

(3) 28 (4) 21

Ans. **(1)**

Sol. $(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 3$

Let

$\Rightarrow x^2 - 9x = t$

$\Rightarrow t^2 + 22t + 121 - t - 20 - 3 = 0$

$\Rightarrow t^2 + 21t + 98 = 0$

$\Rightarrow (t + 14)(t + 7) = 0$

$\Rightarrow t = -7, -14$

So, $x^2 - 9x = -7, -14$

$x^2 - 9x + 7 = 0$ or $x^2 - 9x + 14 = 0$

$$x=\frac{9\pm\sqrt{81-4(7)}}{2\times 1} \qquad x=\frac{9\pm\sqrt{81-4(14)}}{2}$$

$$=\frac{9\pm\sqrt{53}}{2} \qquad =\frac{9\pm 5}{2}$$

Product of all rational roots = 7 × 2 = 14

Option (1)

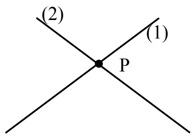
18. Let the line passing through the points (–1, 2, 1) and parallel to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ intersect the line $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}$ at the point P. Then the distance of P from the point Q(4, – 5, 1) is :

(1) 5 (2) 10

(3) $5\sqrt{6}$ (4) $5\sqrt{5}$

Ans. (4)

Sol. Equation of line through point (–1, 2,1) is →



$\Rightarrow \frac{x+1}{2}=\frac{y-2}{3}=\frac{z-1}{4}$ – (2) = λ

So, $\begin{bmatrix} x=2\lambda-1 \\ y=3\lambda+2 \\ z=4\lambda+1 \end{bmatrix}$

By (1) → $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}=\mu$ (Let)

So, $\begin{bmatrix} x=3\mu-2 \\ y=2\mu+3 \\ z=\mu+4 \end{bmatrix}$

For intersection point 'P'

$x = 2\lambda - 1 = 3\mu - 2$

$y = 3\lambda + 2 = 2\mu + 3$ $\begin{bmatrix} \lambda=1 \\ \mu=1 \end{bmatrix}$

$z = 4\lambda + 1 = \mu + 4$

So, point P(x, y, z) = (1, 5, 5)

& Q(4, – 5, 1)

$\therefore PQ = \sqrt{9+100+16}$

$= \sqrt{125} = 5\sqrt{5}$

Option (4)

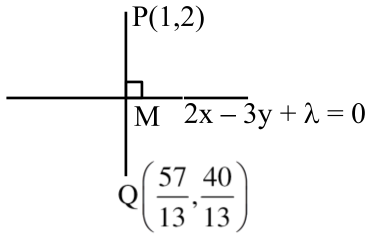
19. Let the lines 3x – 4y – α = 0, 8x – 11y – 33 = 0, and 2x – 3y + λ = 0 be concurrent. If the image of the point (1, 2) in the line 2x – 3y + λ = 0 is $\left(\frac{57}{13},\frac{-40}{13}\right)$, then |αλ| is equal to :

(1) 84 (2) 91

(3) 113 (4) 101

Ans. (2)

Sol.



∵ PM = QM

So, $M\left(\frac{\frac{57}{13}+1}{2},\frac{\frac{-40}{13}+2}{2}\right)$

$=\left(\frac{35}{13},\frac{-7}{13}\right)$

∵ M lies on the time

2x – 3y +λ = 0

$2\left(\frac{35}{13}\right)-3\left(\frac{-7}{13}\right)+\lambda=0$

$\lambda=-\frac{70}{13}+\frac{21}{13}$

$=\frac{-91}{13}=-7$

$\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & 3 & \lambda \end{vmatrix}=0$

⇒ 3(–11λ –99) + 4 (8λ + 66) – α(–24 + 22) = 0

⇒ 33λ – 297 + 32λ + 264 + 24α –22α = 0

⇒ –λ + 2α – 33 = 0 …..(1)

∴ λ = – 7

– (–7) + 2α – 33 = 0

2α = 26

α = 13

∴ |αλ| = |13 ×(–7)|

= 91

20. If the system of equations

2x – y + z = 4

5x + λy + 3z = 12

100 x – 47 y + μz = 212,

has infinitely many solutions, then μ – 2λ is equal to

(1) 56 (2) 59

(3) 55 (4) 57

Ans. (4)

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$

$2(\lambda\mu+141) + (5\mu - 300) - 235 - 100\lambda = 0 \ldots(1)$

$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$

$6\lambda = -12 \Rightarrow \lambda = -2$

Put $\lambda = 2$ in (1)

$2(-2\mu + 141) + 5\mu - 300 - 235 + 200 = 0$

$\mu = 53$

$\therefore$ 57

SECTION-B

21. Let f be a differentiable function such that

$2(x + 2)^2 f(x) - 3(x + 2)^2 = 10 \int_0^x (t+2)f(t)dt$,

$x \geq 0$. Then f(2) is equal to _____.

Ans. **(19)**

Sol. Differentiate both sides

$4(x+2) f(x) + 2(x+2)^2 f'(x) - 6(x+2) = 10(x+2) f(x)$

$2(x+2)^2 f'(x) - 6(x+2)f(x) = 6(x+2)$

$(x+2) \frac{dy}{dx} - 3y = 3$

$\int \frac{dy}{dx} = 3\int \frac{dx}{x+2}$

$\ln (y+1) = 3 \ln(x+2) + C$

$(y + 1) = C(x+2)^3$

$f(0) = \frac{3}{2}$

$f(2) = 19$

22. If for some α, β; $\alpha \leq \beta$, $\alpha + \beta = 8$ and $\sec^2(\tan^{-1}\alpha) + \operatorname{cosec}^2(\cot^{-1}\beta) = 36$, then $\alpha^2 + \beta$ is ______.

Ans. **(14)**

Sol. If$(\tan(\tan^{-1}(\alpha)) + 1 (\cot(\cot^{-1}\beta))^2 = 36$

$\alpha^2 + \beta^2 = 34$

$\alpha\beta = 15$

$\alpha = 3, \beta = 5$

$\therefore \alpha^2 + \beta = 9 + 5 = 14$

23. The number of 3-digit numbers, that are divisible by 2 and 3, but not divisible by 4 and 9, is

Ans. **(125)**

Sol. No, of 3 digits = 999 – 99 = 900

No. of 3 digit numbers divisible by 2 & 3 i.e. by 6

$\frac{900}{6} = 150$

No. of 3 digit numbers divisible by 4 & 9 i.e. by 36

$\frac{900}{36} = 25$

$\therefore$ No of 3 digit numbers divisible by 2 & 3 but not by 4 & 9

150 – 25 = 125

24. Let be a 3 × 3 matrix such that $X^T AX = O$ for all nonzero 3 × 1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

If $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and

$\det (\operatorname{adj}(2(A + I))) = 2^\alpha 3^\beta 5^\gamma$, $\alpha, \beta, \gamma, \in N$, then $\alpha^2 + \beta^2 + \gamma^2$ is

Ans. **(44)**

Sol. $X^TAX = 0$

$$(xyz)\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$(xyz)\begin{pmatrix} a_1x + a_2y + a_3z \\ b_1x + b_2y + b_3z \\ c_1x + c_2y + c_3z \end{pmatrix} = 0$$

$x(a_1x + a_2y + a_3z) + y(b_1x + b_2y + b_3z)$

$+ z(c_1x + c_2y + c_3z) = 0$

$a_1 = 0, b_2 = 0\ c_3 = 0$

$a_2 + b_1 = 0, a_3 + c_1 = 0, b_3 = c_2 = 0$

A = skew symm matrix

$$A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix} \;;\quad A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$x + y = 1$

$-x + z = 4$

$y + z = 5$

$$\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$2x + y = 0 \qquad x = -1$

$-x + z = 4 \qquad y = 2$

$-y - 2z = -8 \qquad z = 3$

$$A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$$

$$2(A+I) = \begin{pmatrix} 2 & -2 & 4 \\ 2 & 2 & 6 \\ -2 & -6 & 2 \end{pmatrix}$$

$2(A+I) = 120 \quad \Rightarrow$ det $|adi(2(A+I))|$

$= 120^2 = 2^6.3^2.5^2$

$\alpha = 6, \beta = 2, \gamma = 2$

25. Let S = {p_1, p_2……, p_{10}} be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct element of S. Then the number of all ordered pairs (x, y), $x \in S$, $y \in A$, such that x divides y , is ______.

Ans. **(5120)**

Sol. Let $\frac{y}{x} = \lambda$

$y = \lambda x$

$= 10 \times ({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + \ldots + {}^9C_9)$

$= 10 \times (2^9)$

10×512

5120

SECTION-A

26. Consider a parallel plate capacitor of area A (of each plate) and separation 'd' between the plates. If E is the electric field and ε_0 is the permittivity of free space between the plates, then potential energy stored in the capacitor is :-

(1) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (2) $\frac{3}{4}\varepsilon_0 E^2 Ad$

(3) $\frac{1}{4}\varepsilon_0 E^2 Ad$ (4) $\varepsilon_0 E^2 Ad$

Ans. (1)

Sol. $\frac{U}{V}=\frac{1}{2}\epsilon_0 E^2$

$U=\frac{1}{2}\epsilon_0 E^2 V$

$=\frac{1}{2}\epsilon_0 E^2 (Ad)$

Ans. (1)

27. What is the relative decrease in focal length of a lens for an increase in optical power by 0.1 D from 2.5 D ? ['D' stands for dioptre]

(1) 0.04 (2) 0.40

(3) 0.1 (4) 0.01

Ans. (1)

Sol. When P = 2.5 D

$F=\frac{1}{P}=\frac{1}{2.5}$

When P' = 2.6 D

$F'=\frac{1}{P'}=\frac{1}{2.6}$

Relative decrease in focal length

$$\frac{F-F'}{F}=\frac{\frac{2}{5}-\frac{5}{13}}{\frac{2}{5}}=1-\frac{25}{26}=\frac{1}{26}=0.04$$

Ans. (1)

28. An air bubble of radius 0.1 cm lies at a depth of 20 cm below the free surface of a liquid of density 1000 kg/m^3. If the pressure inside the bubble is 2100 N/m^2 greater than the atmospheric pressure, then the surface tension of the liquid in SI unit is (use g = 10 m/s^2)

(1) 0.02 (2) 0.1

(3) 0.25 (4) 0.05

Ans. (4)

Sol. T is surface tension

P in air bubble = $P_0+\rho gh+\frac{2T}{R}$

$P_{in}-P_0=\rho gh+\frac{2T}{R}=2100$

$\frac{2T}{R}=2100-\rho gh$

$T=\frac{R}{2}\left(2100-10^3\times 10\times 0.2\right)$

$=\frac{1}{20}\left(2100-2000\right)\times 10^{-2}$

$= 0.05$

Ans. (4)

29. For an experimental expression $y=\frac{32.3\times 1125}{27.4}$, where all the digits are significant. Then to report the value of y we should write :-

(1) y = 1326.2 (2) y = 1326.19

(3) y = 1326.186 (4) y = 1330

Ans. (4)

Sol. $y=\frac{32.3\times 1125}{27.4}=1326.186$

Last significant digits are 3 in operands so results should rounded off to 3 digits.

$\therefore$ y = 1330

Ans. (4)

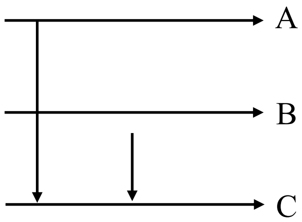
30. During the transition of electron from state A to state C of a Bohr atom, the wavelength of emitted radiation is 2000 Å and it becomes 6000 Å when the electron jumps from state B to state C. Then the wavelength of the radiation emitted during the transition of electrons from state A to state B is :-

(1) 3000 Å (2) 6000 Å

(3) 4000 Å (4) 2000 Å

Ans. **(1)**

Sol.



$E_A - E_C = \frac{hc}{2000Å}$ ….. (i)

and $E_B - E_C = \frac{hc}{6000Å}$ ….. (ii)

Now $E_A - E_B = (E_A - E_C) - (E_B - E_C)$

$\frac{hc}{\lambda_{AB}} = \frac{hc}{2000} - \frac{hc}{6000}$

$\frac{1}{\lambda_{AB}} = \frac{1}{3000Å}$

$\lambda_{AB} = 3000Å$

Ans. (1)

31. Consider the following statements :

A. The junction area of solar cell is made very narrow compared to a photo diode.

B. Solar cells are not connected with any external bias.

C. LED is made of lightly doped p-n junction.

D. Increase of forward current results in continuous increase of LED light intensity.

E. LEDs have to be connected in forward bias for emission of light.

(1) B, D, E Only (2) A, C Only

(3) A, C, E Only (4) B, E Only

Ans. **(4)**

Sol. Conceptual

Ans. (4)

32. The amount of work done to break a big water drop of radius 'R' into 27 small drops of equal radius is 10 J. The work done required to break the same big drop into 64 small drops of equal radius will be :-

(1) 15 J (2) 10 J

(3) 20 J (4) 5 J

Ans. **(1)**

Sol. $W = \Delta U = S\Delta A$

One drop to n drop

$\frac{4}{3}\lambda R^3 = n\frac{4}{3}\lambda r^3$

$r = \frac{R}{n^{\frac{1}{3}}}$

So $W = S(n4\pi r^2 - 4\pi R^2)$

$= S4\pi R^2\left(n^{\frac{1}{3}} - 1\right)$

For on drop to 27 drops

$W = S4\pi R^2\left(27^{\frac{1}{3}} - 1\right) = 10$ ….. (i)

For one drop to 64 drops

$W' = S4\pi R^2\left(64^{\frac{1}{3}} - 1\right)$ … (ii)

(ii)/(i)

$\frac{W'}{W} = \frac{4-1}{3-1} = \frac{3}{2}$

$W' = \frac{3}{2}W = 155$

Ans. (1)

33. An object of mass 'm' is projected from origin in a vertical xy plane at an angle 45° with the x-axis with an initial velocity v_0. The magnitude and direction of the angular momentum of the object with respect to origin, when it reaches at the maximum height, will be [g is acceleration due to gravity]

(1) $\frac{mv_0^3}{2\sqrt{2}g}$ along negative z-axis

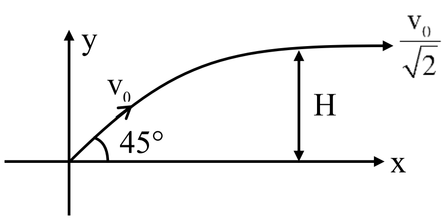
(2) $\frac{mv_0^3}{2\sqrt{2}g}$ along positive z-axis

(3) $\frac{mv_0^3}{4\sqrt{2}g}$ along positive z-axis

(4) $\frac{mv_0^3}{4\sqrt{2}g}$ along negative z-axis

Ans. (4)

Sol.



$$H = \frac{\left(\frac{v_0}{\sqrt{2}}\right)^2}{2g} = \frac{v_0^2}{4g}$$

L = mvh

$$L = m\frac{v_0}{\sqrt{2}}\frac{v_0^2}{4g}$$

Ans. (4)

34. The Young's double slit interference experiment is performed using light consisting of 480 nm and 600 nm wavelengths to form interference patterns. The least number of the bright fringes of 480 nm light that are required for the first coincidence with the bright fringes formed by 600 nm light is :-

(1) 4 (2) 8

(3) 6 (4) 5

Ans. (4)

Sol. $\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

n 480 = m 600

$n_{min} = 5$

Ans. (4)

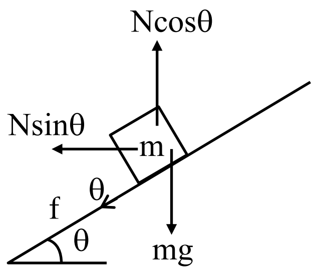
35. A car of mass 'm' moves on a banked road having radius 'r' and banking angle θ. To avoid slipping from banked road, the maximum permissible speed of the car is v_0. The coefficient of friction μ between the wheels of the car and the banked road is :-

(1) $\mu = \frac{v_0^2 + rg\tan\theta}{rg - v_0^2\tan\theta}$ (2) $\mu = \frac{v_0^2 + rg\tan\theta}{rg + v_0^2\tan\theta}$

(3) $\mu = \frac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}$ (4) $\mu = \frac{v_0^2 - rg\tan\theta}{rg - v_0^2\tan\theta}$

Ans. (3)

Sol.



$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

$N\cos\theta - f\sin\theta = mg$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{Rg}$$

$Rg\tan\theta + \mu Rg = v^2 - v^2\mu\tan\theta$

$$\mu = \frac{v^2 - Rg\tan\theta}{Rg + v^2\tan\theta}$$

Ans. (3)

36. A uniform solid cylinder of mass 'm' and radius 'r' rolls along an inclined rough plane of inclination 45°. If it starts to roll from rest from the top of the plane then the linear acceleration of the cylinder axis will be :-

(1) $\frac{1}{\sqrt{2}}g$ (2) $\frac{1}{3\sqrt{2}}g$

(3) $\frac{\sqrt{2}g}{3}$ (4) $\sqrt{2}g$

Ans. (3)

Sol. $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$

$$a = \frac{\frac{g}{\sqrt{2}}}{1+\frac{1}{2}} = \frac{2\frac{g}{\sqrt{2}}}{3} = \frac{\sqrt{2}g}{3}$$

Ans. (3)

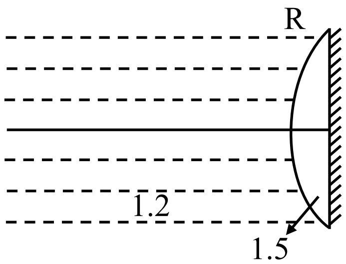
37. A thin plano convex lens made of glass of refractive index 1.5 is immersed in a liquid of refractive index 1.2. When the plane side of the lens is silver coated for complete reflection, the lens immersed in the liquid behaves like a concave mirror of focal length 0.2 m. The radius of curvature of the curved surface of the lens is :-

(1) 0.15 m (2) 0.10 m

(3) 0.20 m (4) 0.25 m

Ans. (2)

Sol.



$$\frac{1.5}{v} = \frac{1.5-1.2}{R}$$

$$v = \frac{1.5R}{0.3} = 5R$$

$$\frac{1.2}{f} - \frac{1.5}{5R} = \frac{1.2-1.5}{-R}$$

$$\frac{1.2}{f} = \frac{0.3}{R}\times 2 \Rightarrow f = 2R \Rightarrow R = 0.1$$

Ans (2)

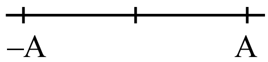
38. A particle is executing simple harmonic motion with time period 2s and amplitude 1 cm. If D and d are the total distance and displacement covered by the particle in 12.5 s, then $\frac{D}{d}$ is :-

(1) $\frac{15}{4}$ (2) 25

(3) 10 (4) $\frac{16}{5}$

Ans. (2)

Sol. A = 1 cm



$$n = \frac{12.5}{2} = 6.25\text{cycles}$$

$\therefore$ D = 4 × 6 + 1 = 25

d = 1

$$\frac{D}{d} = 25$$

Ans. (2)

39. A satellite is launched into a circular orbit of radius 'R' around the earth. A second statellite is launched into an orbit of radius 1.03 R. The time period of revolution of the second satellite is larger than the first one approximately by :-

(1) 3 % (2) 4.5 %

(3) 9 % (4) 2.5 %

Ans. (2)

Sol. $T^2 = KR^3$

$$\frac{2\Delta T}{T} = \frac{3\Delta R}{R}$$

$$\frac{2\Delta T}{T} = \frac{3\times 0.03R}{R}$$

$$\frac{\Delta T}{T} = \frac{3\times 0.03}{2}\times 100 = 4.5\%$$

Ans. (2)

40. A plano-convex lens having radius of curvature of first surface 2 cm exhibits focal length of f_1 in air. Another plano-convex lens with first surface radius of curvature 3 cm has focal length of f_2 when it is immersed in a liquid of refractive index 1.2. If both the lenses are made of same glass of refractive index 1.5, the ratio of f_1 and f_2 will be :-

(1) 3 : 5 (2) 1 : 3

(3) 1 : 2 (4) 2 : 3

Ans. (2)

Sol. $\frac{1}{f_1} = (1.5-1)\left[\frac{1}{2}-0\right] \Rightarrow f_1 = 4\text{cm}$

$$\frac{1}{f_2} = \left(\frac{1.5}{1.2}-1\right)\left(\frac{1}{3}-0\right)$$

$$\frac{1}{f_2} = \frac{0.3}{1.2}\times\frac{1}{3}$$

$f_2 = 12$

$f_1 : f_2 = 4 : 12 = 1 : 3$

Ans. (2)

41. An alternating current is given by

$I = I_A \sin \omega t + I_B \cos \omega t$.

The r.m.s. current will be :-

(1) $\sqrt{I_A^2 + I_B^2}$ (2) $\frac{\sqrt{I_A^2 + I_B^2}}{2}$

(3) $\sqrt{\frac{I_A^2 + I_B^2}{2}}$ (4) $\frac{|I_A + I_B|}{\sqrt{2}}$

Ans. (3)

Sol. $i_{rms} = \sqrt{\frac{\int I^2 dt}{\int dt}}$

$$\sqrt{\frac{I_A^2 + I_B^2}{2}} = i_{rms}$$

42. An electron of mass 'm' with an initial velocity $\vec{v} = v_0\hat{i}\,(v_0 > 0)$ enters an electric field $\vec{E} = -E_0\hat{k}$. If the initial de Broglie wavelength is λ_0, the value after time t would be :-

(1) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$ (2) $\frac{\lambda_0}{\sqrt{1-\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

(3) λ_0 (4) $\lambda_0\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}$

Ans. (1)

Sol. $\vec{v} = v_0\hat{i} - \frac{E_0 e}{m}t\hat{k}$

$$|\vec{v}| = \sqrt{v_0^2 + \frac{E_0^2e^2t^2}{m^2}}$$

$$\lambda_0 = \frac{h}{mv_0}$$

$$\lambda' = \frac{h}{mv_0\sqrt{1+\frac{E_0^2e^2t^2}{v_0^2m^2}}}$$

$$\lambda' = \frac{\lambda_0}{\sqrt{1+\frac{E_0^2e^2t^2}{v_0^2m^2}}}$$

Ans. (1)

43. A parallel plate capacitor was made with two rectangular plates, each with a length of l = 3 cm and breath of b = 1 cm. The distance between the plates is 3 μm. Out of the following, which are the ways to increase the capacitance by a factor of 10 ?

A. l = 30 cm, b = 1 cm, d = 1 μm

B. l = 3 cm, b = 1 cm, d = 30 μm

C. l = 6 cm, b = 5 cm, d = 3 μm

D. l = 1 cm, b = 1 cm, d = 10 μm

E. l = 5 cm, b = 2 cm, d = 1 μm

Choose the correct answer from the options given below :

(1) C and E only (2) B and D only

(3) A only (4) C only

Ans. **(1)**

Sol. $C = \frac{A\in_0}{d}$

A : plate area

d : distance between the plates.

Capacitance initial

$= \frac{\in_0 \ell b}{d} = \in_0$ units

Option 'C' $\ell = 6$ cm

$b = 5$ cm

$d = 3$ cm

Capacitance = 10 $\in_0$ units

Option 'E' $\ell = 5$ cm

$b = 2$ cm

$d = 1$ cm

Capacitance = 10 $\in_0$ units

$\therefore$ Ans is option (1)

44. A force $F = \alpha + \beta x^2$ acts on an object in the x-direction. The work done by the force is 5J when the object is displaced by 1 m. If the constant $\alpha = 1$N then β will be

(1) 15 N/m^2 (2) 10 N/m^2

(3) 12 N/m^2 (4) 8 N/m^2

Ans. **(3)**

Sol. $F = \alpha + \beta x^2$

Work done $= \int F dx$

$$5 = \int (\alpha + \beta x^2) dx$$

$$5 = \alpha x + \left.\frac{\beta x^3}{3}\right|_0^1$$

$$5 = \alpha + \frac{\beta}{3}[\alpha = 1]$$

$$4 = \frac{\beta}{3} \Rightarrow \beta = 12 N/m^2$$

45. An ideal gas goes from an initial state to final state. During the process, the pressure of gas increases linearly with temperature.

A. The work done by gas during the process is zero.

B. The heat added to gas is different from change in its internal energy.

C. The volume of the gas is increased.

D. The internal energy of the gas is increased.

E. The process is isochoric (constant volume process)

Choose the ***correct*** answer from the options given below :-

(1) A, B, C, D Only

(2) A, D, E Only

(3) E Only

(4) A, C Only

Ans. **(2)**

Sol. Given that

$P = kT$

$\frac{P}{T}$ = constant

$\therefore$ Volume is constant or isochoric process.

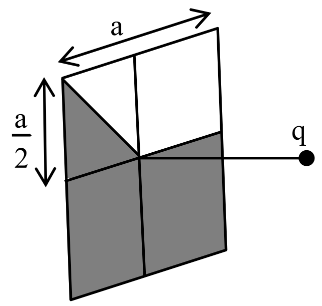
$\therefore W_D = 0$

$\therefore Q = \Delta U$

Also temperature increases hence internal energy increases.

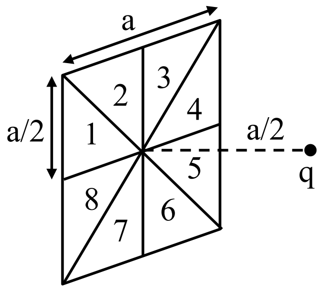
SECTION-B

46. A square loop of sides a = 1 m is held normally in front of a point charge q = 1C. The flux of the electric field through the shaded region is $\frac{5}{p}\times\frac{1}{\varepsilon_0}\frac{Nm^2}{C}$, where the value of p is ____.



Ans. (48)

Sol.



Total flux through square $=\frac{q}{\epsilon_0}\left(\frac{1}{6}\right)$

Lets divide square is 8 equal parts.

Flux is same for each part.

$\therefore$ Flux through shaded portion is $\frac{5}{8}$ (Total flux)

$$=\frac{5}{8}\times\frac{q}{\epsilon_0}\frac{1}{6}=\frac{5}{48}\frac{1}{\epsilon_0}$$

$\therefore$ required Ans. is 48

Note : Distnace of charge from square loop is not mentioned we have assume it as $\frac{a}{2}$.

47. The least count of a screw guage is 0.01 mm. If the pitch is increased by 75% and number of divisions on the circular scale is reduced by 50%, the new least count will be _____ $\times 10^{-3}$ mm.

Ans. (35)

Sol. Given least count of Screw Gauge = 0.01 mm

$$L.C=\frac{(\text{pitch})}{\text{No. of circular turn}}=\frac{P}{N}=0.01\text{ mm}$$

$$\text{New pitch}=\frac{P(1+0.75)}{N(1-0.5)}=\frac{P}{N}\left[\frac{1.75}{0.5}\right]$$

= (0.01) 3.5

= 0.035 mm

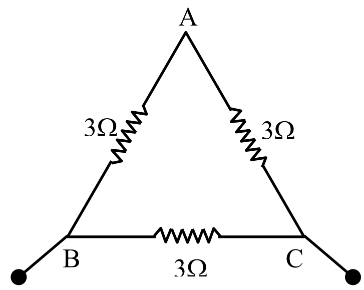
= 35×10^{-3} mm

$\therefore$ Ans. is 35

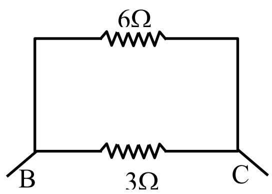
48. A wire of resistance 9 Ω is bent to form an equilateral triangle. Then the equivalent resistance across any two vertices will be __ohm.

Ans. (2)

Sol.



⇒



9Ω is the resistance of whole wire

$\therefore$ resistance of each wire = 3Ω.

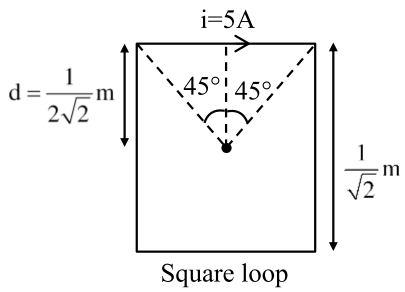
$\therefore$ Equivalent resistance = 2Ω

49. A current of 5A exists in a square loop of side $\frac{1}{\sqrt{2}}$ m. Then the magnitude of the magnetic field B at the centre of the square loop will be $p \times 10^{-6}$ T. where, value of p is ______.

[Take $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1}].

Ans. **(8)**

Sol.



Let B be the magnetic field due to single side

then $B = \frac{\mu_0 i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

$$= \frac{10^{-7} \times 5 \times 2}{\frac{1}{2\sqrt{2}}} \times \frac{1}{\sqrt{2}} = 2 \times 10^{-6}$$

$\therefore B_{net}$ at centre O = 4B

$= 8 \times 10^{-6}$

$\therefore$ P = 8 Ans.

50. The temperature of 1 mole of an ideal monoatomic gas is increased by 50°C at constant pressure. The total heat added and change in internal energy are E_1 and E_2, respectively. If $\frac{E_1}{E_2} = \frac{x}{9}$ then the value of x is _______.

Ans. **(15)**

Sol. Given that process is isobaric $\Delta T = 50°C$

Q in isobaric process = $nC_p\Delta T = E_1$

ΔU in isobaric process = $nC_v\Delta T = E_2$

$\therefore \frac{E_1}{E_2} = \frac{C_P}{C_V} = \gamma$

Given, gas is monoatomic

$\therefore \gamma = 1 + \frac{2}{f}$

$= 1 + \frac{2}{3}$

$= \frac{5}{3}$

Now, as per question.

$\frac{5}{3} = \frac{x}{9}$

x = 15

SECTION-A

51. For the given cell

$Fe^{2+}(eq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

The standard cell potential of the above reaction is

Given :

$Ag^{+} + e^{-} \rightarrow Ag \qquad E^{0} = xV$

$Fe^{2+} + 2e^{-} \rightarrow Fe \qquad E^{0} = y\ V$

$Fe^{3+} + 3e^{-} \rightarrow Fe \qquad E^{0} = zV$

(1) $x + y - z$

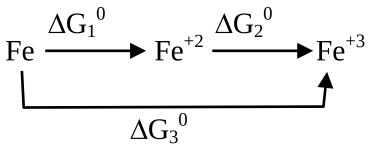
(2) $x + 2y - 3z$

(3) $y - 2x$

(4) $x + 2y$

Ans. (2)

Sol. $Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$



$\Delta G_3^0 = \Delta G_1^0 + \Delta G_2^0$

$-3F(-z) = -2F(-y) + \Delta G_2^0$

$\Delta G_2^0 = 3Fz - 2Fy$

Also $\Delta G_2^0 = -nFE^0_{Fe^{+2}/Fe^{+3}}$

$3Fz - 2Fy = -1F(E^0_{Fe^{+2}/Fe^{+3}})$

$E^0_{Fe^{+2}/Fe^{+3}} = 2y - 3z$

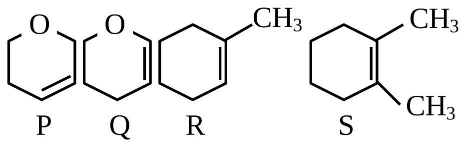
E^0_{Cell} for reaction will be

$E^0_{Ag^{+}/Ag} + E^0_{Fe^{+2}/Fe^{+3}}$

$= x + 2y - 3z$

Option (2)

52. Following are the four molecules "P", "Q", "R" and "S".
Which one among the four molecules will react with H-Br(aq) at the fastest rate ?

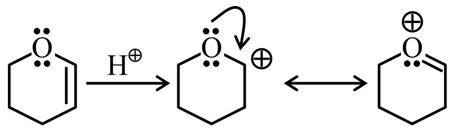


P Q R S

(1) S (2) Q
(3) R (4) P

Ans. (2)

Sol. Addition of $H\text{-}Br_{(aq)}$ to alkene follows electrophilic addition mechanism. In the rate determining step a carbocation intermediate is formed. Among P, Q, R & S compound Q will form most stable carbocation intermediate since it is resonance stabilized.



53. One mole of the octahedral complex compound $Co(NH_3)_5Cl_3$ gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with excess of $AgNO_3$ solution to yield two moles of $AgCl_{(s)}$. The structure of the complex is :

(1) $[Co(NH_3)_5Cl]Cl_2$

(2) $[Co(NH_3)_4Cl].Cl_2.NH_3$

(3) $[Co(NH_3)_4Cl_2]Cl.NH_3$

(4) $[Co(NH_3)_3Cl_3].2NH_3$

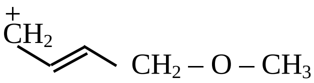
Ans. (1)

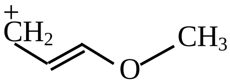
Sol.

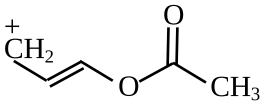
$$[Co(NH_3)_5Cl]Cl_2 \rightarrow \underbrace{[Co(NH_3)_5Cl]^{2+}(aq.) + 2Cl^{-}(aq.)}_{\text{3 ions in water}}$$

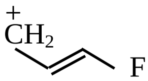
$[Co(NH_3)_5Cl]Cl_2(aq.) + 2AgNO_3(aq.) \rightarrow$
$[Co(NH_3)_5Cl](NO_3)_2(aq.) + 2AgCl(s)$

54. Which one of the carbocations from the following is most stable ?

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. Carbocation intermediate is stabilised by +I, +M & hyperconjugation effect. Since in option 2 carbocation is in conjugation with stronger +M group $-OCH_3$ hence it will be most stable.

55. Which of the following linear combination of atomic orbitals will lead to formation of molecular orbitals in homonuclear diatomic molecules [internuclear axis in z-direction] ?

A. $2p_z$ and $2p_x$

B. $2s$ and $2p_x$

C. $3d_{xy}$ and $3d_{x^2-y^2}$

D. $2s$ and $2p_z$

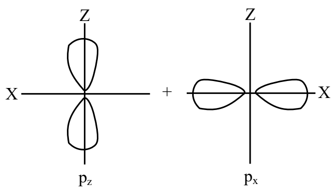
E. $2p_z$ and $3d_{x^2-y^2}$

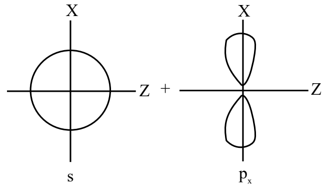
(1) E Only (2) A and B Only

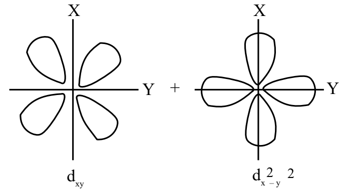
(3) D Only (4) C and D Only

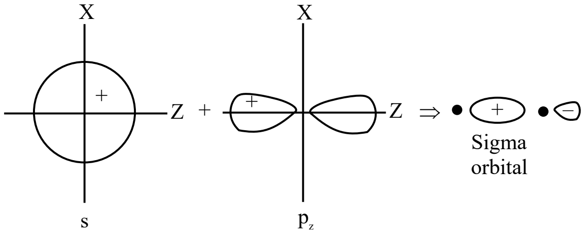
Ans. **(3)**

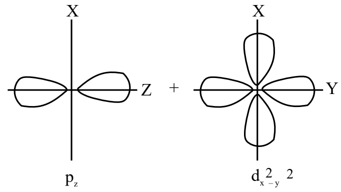
Sol.

A :  ⇒ no molecular orbital

B :  ⇒ no molecular orbital

C :  ⇒ no molecular orbital

D : 

E :  ⇒ no molecular orbital

56. Which of the following ions is the strongest oxidizing agent ?

(Atomic Number of Ce = 58, Eu = 63, Tb = 65, Lu = 71]

(1) Lu^{3+} (2) Eu^{2+}

(3) Tb^{4+} (4) Ce^{3+}

Ans. **(3)**

Sol. Tb^{4+} is strongest oxidising agent

57. Ksp for $Cr(OH)_3$ is 1.6×10^{-30}. What is the molar solubility of this salt in water?

(1) $\sqrt[4]{\dfrac{1.6\times10^{-30}}{27}}$ (2) $\dfrac{1.8\times10^{-30}}{27}$

(3) $\sqrt[5]{1.8\times10^{-30}}$ (4) $\sqrt[2]{1.6\times10^{-30}}$

Ans. **(1)**

Sol. $Cr(OH)_{3\,(s)} \rightleftharpoons Cr^{+3}_{(aq)} + 3OH^-_{(aq)}$

At eq : s 3s

$K_{sp} = (s).(3s)^3 = 27s^4$

$27s^4 = 1.6 \times 10^{-30}$

$s = \left(\dfrac{1.6}{27}\times10^{-30}\right)^{1/4}$

Option (1)

58. Let us consider an endothermic reaction which is non-spontaneous at the freezing point of water. However, the reaction is spontaneous at boiling point of water. Choose the correct option.

(1) Both ΔH and ΔS are (+ve)

(2) ΔH is (–ve) but ΔS is (+ve)

(3) ΔH is (+ve) but ΔS is (–ve)

(4) Both ΔH and ΔS are (–ve)

Ans. **(1)**

Sol. Reaction is spontaneous at relatively high temperature and non-spontaneous at low temperature $\Delta G = \Delta H - T\Delta S$

It is only possible when ΔH and ΔS both are positive.

Option (1)

59. Given below are two statements I and II.

Statement I : Dumas method is used for estimation of "Nitrogen" in an organic compound.

Statement II : Dumas method involves the formation of ammonium sulphate by heating the organic compound with conc H_2SO_4.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Ans. **(4)**

Sol. In Dumas method nitrogen present in organic compound is converted into N_2 gas whose volumetric analysis gives the percentage of nitrogen atom in the organic compound.

60. Which of the following Statements are NOT true about the periodic table?

A. The properties of elements are function of atomic weights.

B. The properties of elements are function of atomic numbers.

C. Elements having similar outer electronic configuration are arranged in same period.

D. An element's location reflects the quantum numbers of the last filled orbital.

E. The number of elements in a period is same as the number of atomic orbitals available in energy level that is being filled.

Choose the correct answer from the options given below:

(1) A, C and E Only (2) D and E Only

(3) A and E Only (4) B, C and E Only

Ans. **(1)**

Sol. Properties of elements are periodic function of their atomic number. Elements having similar outer electronic configuration are arranged in same group. Number of elements in a period is not equal to number of atomic orbitals available in energy level that is being filled.

Hence, A, C & E are incorrect

61. The carbohydrates "Ribose" present in DNA, is

A. A pentose sugar

B. present in pyranose from

C. in "D" configuration

D. a reducing sugar, when free

E. in α-anomeric form

Choose the correct answer from the options given below :

(1) A, C and D Only

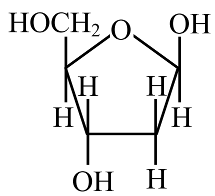
(2) A, B and E Only

(3) B, D and E Only

(4) A, D and E Only

Ans. **(1)**

Sol. In Ribose carbohydrate present in DNA is β-2-Deoxy-D-Ribose whose structure is



which is a reducing D-sugar in β anomeric form & it is a pentose sugar.

62. Preparation of potassium permanganate from MnO_2 involves two step process in which the 1st step is a reaction with KOH and KNO_3 to produce

(1) $K_4[Mn(OH)_6]$ (2) K_3MnO_4

(3) $KMnO_4$ (4) K_2MnO_4

Ans. **(4)**

Sol. $MnO_2 \xrightarrow[KNO_3,\Delta]{KOH} K_2MnO_4$

63. The large difference between the melting and boiling points of oxygen and sulphur may be explained on the basis of

(1) Atomic size (2) Atomicity

(3) Electronegativity (4) Electron gain enthalpy

Ans. **(2)**

Sol. Oxygen exists as O_2 (Atomicity = 2)

Sulphur exists as S_8 (Atomicity = 8)

Hence, Melting point & Boiling point of sulphur are significantly large compared to oxygen.

64. For a reaction, $N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2} O_{2(g)}$ in a constant volume container, no products were present initially. The final pressure of the system when 50% of reaction gets completed is

(1) 7/2 times of initial pressure

(2) 5 times of initial pressure

(3) 5/2 times of initial pressure

(4) 7/4 times of initial pressure

Ans. **(4)**

Sol

	$N_2O_{5(g)} \longrightarrow$	$2NO_{2(g)} +$	$\frac{1}{2}O_{2(g)}$
t = 0	P_0	–	–
t = t	$P_0 - x$	2x	$\frac{x}{2}$

$$x = \frac{P_0}{2}$$

$$P_{total} = P_0 - \frac{P_0}{2} + P_0 + \frac{P_0}{4} = \frac{7}{4}P_0$$

Option (4)

65. Which of the following arrangements with respect to their reactivity in nucleophilic addition reaction is correct?

(1) benzaldehyde < acetophenone < p-nitrobenzaldehyde < p-tolualdehyde

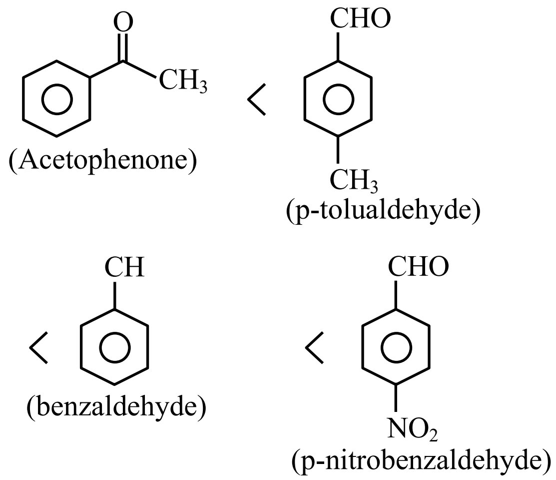
(2) acetophenone < benzaldehyde < p-tolualdehyde < p-nitrobenzaldehyde

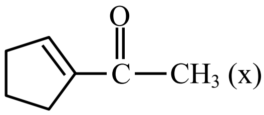
(3) acetophenone < p-tolualdehyde < benzaldehyde < p-nitrobenzaldehyde

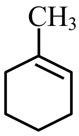
(4) p-nitrobenzaldehyde < benzaldehyde < p-tolualdehyde < acetophenone

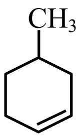
Ans. **(3)**

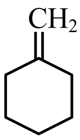
Sol. The rate of nucleophilic addition decreased due to steric crowding around carbonyl carbon & increased by electron withdrawing group if the steric crowding is same hence the reactivity towards nucleophilic addition will be

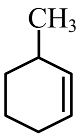


66. Aman has been asked to synthesise the molecule  . He thought of preparing the molecule using an aldol condensation reaction. He found a few cyclic alkenes in his laboratory. He thought of performing ozonolysis reaction on alkene to produce a dicarbonyl compound followed by aldol reaction to prepare "x". Predict the suitable alkene that can lead to the formation of "x".

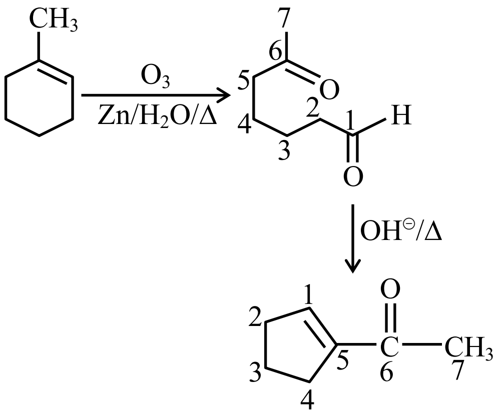
(1) 

(2) 

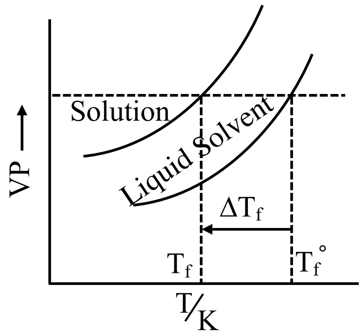
(3) 

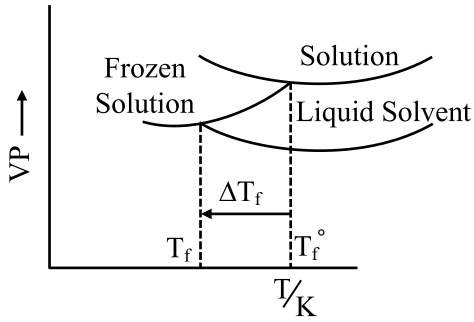
(4) 

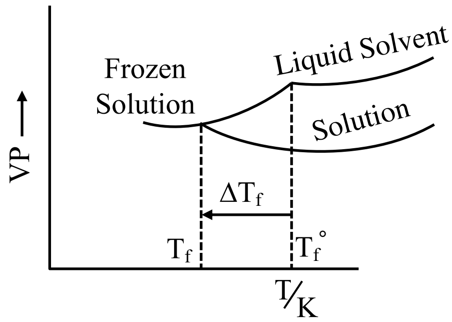
Ans. **(1)**

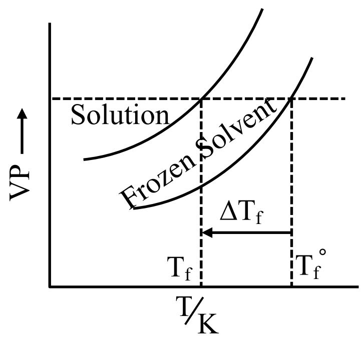
Sol. 

67. Consider the given plots of vapour pressure (VP) vs temperature (T/K) Which amongst the following options is correct graphical representation showing ΔT_f, depression in the freezing point of solvent in a solution ?

(1) 

(2) 

(3) 

(4) 

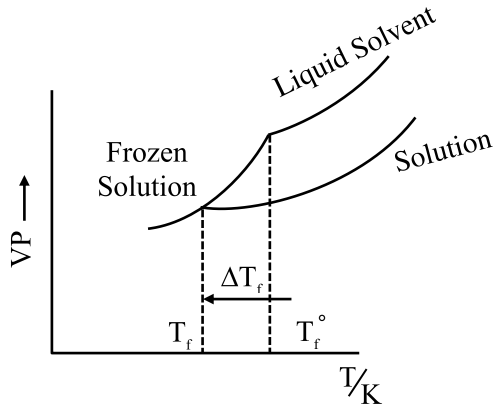
Ans. **(3)**

Sol. On adding non-volatile solute in a solvent, the freezing point of solution decreases.

$T_f < T_f^0$

F.P. of solution < F.P. of pure solvent

Also V.P. of solution decreases on adding non-volatile solute in a solvent.



68. Which of the following statement is true with respect to H_2O, NH_3 and CH_4?

A. The central atoms of all the molecules are sp^3 hybridized.

B. The H–O–H, H–N–H and H–C–H angles in the above molecules are 104.5º, 107.5º and 109.5º respectively.

C. The increasing order of dipole moment is $CH_4 < NH_3 < H_2O$.

D. Both H_2O and NH_3 are Lewis acids and CH_4 is a Lewis base

E. A solution of NH_3 in H_2O is basic. In this solution NH_3 and H_2O act as Lowry-Bronsted acid and base respectively.

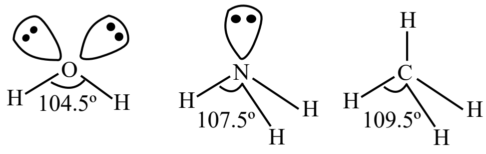
Choose the correct answer from the options given below :

(1) A, B and C only (2) C, D and E only

(3) A, D and E only (4) A, B, C and E only

Ans. (1)

Sol.



$O \rightarrow sp^3$ $N \rightarrow sp^3$ $C \rightarrow sp^3$

Dipole moment $H_2O > NH_3 > CH_4$

H_2O & NH_3 are Lewis Bases

NH_3 act as Lowry- Bronsted base

Hence, A, B & C are correct

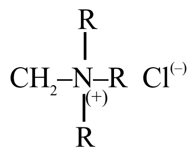
69. Given below are two statements :

Statement-I : The conversion proceeds well in the less polar medium.

$CH_3–CH_2–CH_2–CH_2–Cl \xrightarrow{HO^-} CH_3–CH_2–CH_2–CH_2–OH + Cl^-$

Statement-II : The conversion proceeds well in the more polar medium.

$CH_3–CH_2–CH_2–CH_2–Cl \xrightarrow{R_3\ddot{N}} CH_3–CH_2–CH_2—$



In the light of the above statements, choose the *correct* answer from the options given below.

(1) Both statement I and statement II are true

(2) Both statement I and statement II are false

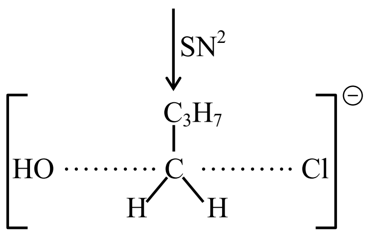
(3) Statement I is false but statement II is true

(4) Statement I is true but statement II is false

Ans. (1)

Sol. $CH_3 – CH_2 – CH_2 – CH_2 – Cl + \overset{\ominus}{O}H$

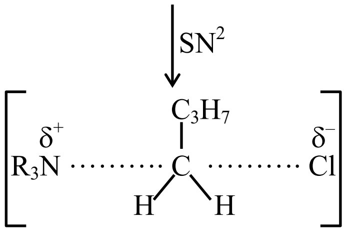
Reactant (higher charge density)



Transition state (less charge density)

⇒ This reaction will proceed faster in less polar medium which will not increase the activation energy value.

$CH_3 - CH_2 - CH_2 - CH_2 - Cl + R_3N$
Reactant (low charge density)

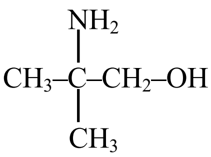


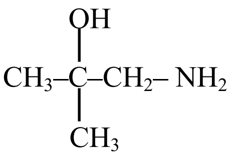
Transition state (Higher charge density)

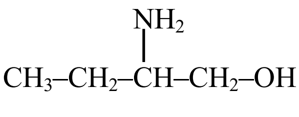
⇒ This reaction will proceed faster in more polar medium which will decrease the activation energy value.

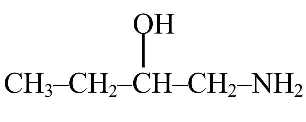
70. The product (A) formed in the following reaction sequence is :

$$CH_3{-}C{\equiv}CH \xrightarrow[\text{(iii)}H_2/Ni]{\text{(i)}Hg^{2+},\ H_2SO_4\ \ \text{(ii)}HCN} \underset{\text{Product}}{(A)}$$

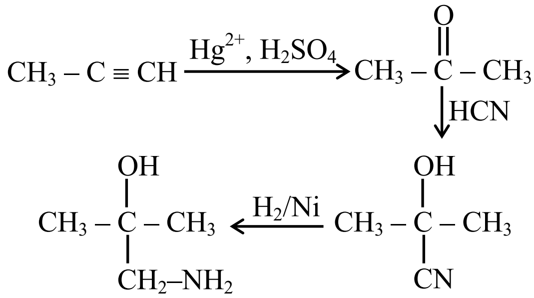
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

SECTION-B

71. 37.8 g N_2O_5 was taken in a 1 L reaction vessel and allowed to undergo the following reaction at 500 K

$2N_2O_{5(g)} \rightarrow 2N_2O_{4(g)} + O_{2(g)}$

The total pressure at equilibrium was found to be 18.65 bar.

Then, Kp = _______ $\times 10^{-2}$ [nearest integer]
Assume N_2O_5 to behave ideally under these conditions
Given : R = 0.082 bar L mol^{-1} K^{-1}

Ans. (962)

Sol. Initial pressure of N_2O_5

$$= \frac{\frac{37.8}{108} \times 0.082 \times 500}{1} = 14.35\,\text{bar}$$

$$2N_2O_5 \rightleftharpoons 2N_2O_4 + O_2$$

t = 0	14.35		
t = eq	14.35 –2P	2P	P

P_{Total} at eqb = 14.35 + P = 18.65
P = 4.3

$P_{N_2O_5} = 5.75\,\text{bar}$

$P_{N_2O_4} = 8.6\,\text{bar}$

$P_{O_2} = 4.3\,\text{bar}$

$$k_p = \frac{(8.6)^2 \times (4.3)}{(5.75)^2} = 9.619 = x \times 10^{-2}$$

$x = 961.9 \approx 962$

Ans. 962

72. Standard entropies of X_2, Y_2 and XY_5 are 70, 50 and 110 J K^{-1} mol^{-1} respectively. The temperature in Kelvin at which the reaction

$$\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightarrow XY_5 \;\; \Delta H^{-} = -35 \text{ kJ mol}^{-1}$$

Will be at equilibrium is ____ (Nearest integer)

Ans. (700)

Sol. $$\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightleftharpoons XY_5$$

$$\Delta S^0_{Rxn} = 110 - \left[\left(\frac{1}{2}\times 70\right) + \left(\frac{5}{2}\times 50\right)\right]$$

$= 110 - 160 = -50$ JK^{-1} mol^{-1}

$\Delta G^0 = 0$ at eqb

$\Delta G^0 = \Delta H^0 - T\Delta S^0$

$0 = -35000 - T(-50)$

T = 700 Kelvin

Ans. 700

73. Xg of benzoic acid on reaction with aq. $NaHCO_3$ release CO_2 that occupied 11.2 L volume at STP. X is _____ g.

Ans. (61)

Sol. $C_6H_5COOH + NaHCO_3 \rightarrow C_6H_5COO^-Na^+ + H_2O + CO_2$
x gm 11.2 L

mole of C_6H_5COOH = mole of CO_2 = $\frac{11.2}{22.4} = 0.5$

mass of C_6H_5COOH = x = 0.5 × 122 = 61 gm

Ans. 61

74. Among the following cations, the number of cations which will give characteristic precipitate in their identification tests with $K_4[Fe(CN)_6]$ is :

Cu^{2+}, Fe^{3+}, Ba^{2+}, Ca^{2+}, NH_4^+, Mg^{2+}, Zn^{2+}

ALELN Ans. (4)

NTA Ans. (3)

Sol. Only Cu^{2+}, Fe^{3+}, Ca^{2+} & Zn^{2+} form precipitate with $K_4[Fe(CN)_6]$

75. Consider the following reaction occurring in the blast furnace.

$Fe_3O_{4(s)} + 4CO_{(g)} \rightarrow 3Fe_{(l)} + 4CO_{2(g)}$

'*x*' kg of iron is produced when 2.32 × 10^3 kg Fe_3O_4 and 2.8 × 10^2 kg CO are brought together in the furnace. The value of 'x' is ____. (nearest integer)

{Given :

Molar mass of Fe_3O_4 = 232 g mol^{-1}

Molar mass of CO = 28 g mol^{-1}

Molar mass of Fe = 56 g mol^{-1}}

Ans. (420)

Sol. moles of $Fe_3O_4 = \frac{2.32\times 10^3 \times 10^3}{232} = 10000$ mol

moles of $CO = \frac{2.8\times 10^2 \times 10^3}{28} = 10000$ mol

$$\underset{10^4 \text{mol}}{Fe_3O_4} + \underset{10^4 \text{mol}}{4CO} \longrightarrow 3Fe + 4CO_2$$

CO is L.R.

mole of Fe = $\frac{3}{4}\times 10^4$

mass of Fe = $\frac{3}{4}\times\frac{10^4\times 56}{1000}$ kg = 420kg

Ans. 420